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A New Metaphor: How Artificial Intelligence Links Legal Reasoning and Mathematical Thinking

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A NEW METAPHOR: HOW ARTIFICIAL INTELLIGENCE LINKS LEGAL REASONING AND MATHEMATICAL THINKING

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> *The lessons of mathematics are simple ones and there are no numbers in them: that there is structure in this world; that we can hope to understand some of it and not just gape at what our senses present to us; that our intuition is stronger with a formal exoskeleton than without one. And that mathematical certainty is one thing, the softer convictions we find attached to us in everyday life another, and we should keep track of the difference if we can.*[1]

Artificial intelligence's (AI's) impact on the legal community expands exponentially each year. As AI advances, lawyers have more powerful tools to enhance their ability to research and analyze the law, as well as to draft contracts and other legal documents. Lawyers are already using tools powered by AI and are learning to shift their methodologies to take advantage of these enhancements. To continue to grow into their shifting role, lawyers should understand the relationship between AI, mathematics, and legal reasoning.

In the realm of law and rhetoric, mathematics has traditionally been used as a metaphor for the idea that formal logical reasoning is at odds with legal analysis. Specifically, in this conventional metaphor, mathematics has been viewed as separate from legal reasoning, because mathematical logic does not consider the human aspects of law such as public policy and custom. And yet, the application of AI, with its underlying mathematical algorithms, presents a paradox, because AI demonstrates that mathematical models do work in tandem with legal reasoning, research, and drafting. Mathematical reasoning is an analog to legal reasoning, exemplified by the very fact that AI can

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1. JORDAN ELLENBERG, HOW NOT TO BE WRONG: THE POWER OF MATHEMATICAL THINKING 437 (2014).

replicate legal research, some aspects of legal reasoning, and even contract drafting. Therefore, we need a new metaphor, one where mathematics represents a standard of rigorous and precise thinking.

The connection between AI and the law may seem contrary to traditional notions that math is incompatible with the law. But this is not to suggest that scholars and legal thinkers have been wrong; the law is not equivalent to mathematics. Rather, this Article suggests that mathematics can complement legal thinking, just as social science can inform public policy arguments. An adept mathematician possesses strong logical reasoning, understanding the structure of an idea such that they can develop principles that remain true even if the structural components change. Logical reasoning is not the end goal of legal reasoning, but it is a crucial component. A lawyer must employ rigorous and precise thinking to test the logic of an argument, just like a mathematician. Furthermore, as AI comes into the forefront of lawyering, a lawyer who understands mathematical thinking will be empowered to harness the power of AI as a legal tool in an ever-advancing world.

I. INTRODUCTION

The legal field's relationship to mathematics resembles that of a child being asked to eat all the veggies on their plate—no one wants to take a bite. This seemingly widespread acceptance of the narrative that lawyers are bad at math can be seen from law students' reactions to the mere thought of calculating damages in a first-year civil procedure course to those of well-known legal thinkers. The "in" joke is that we are all bad at math, and we became lawyers because we are bad at math.[2] Even Chief Justice John Roberts joked, "I think there are a lot of people who go to law school because they're not good at math and can't think of anything else to do."[3] Or take, for example, Former First Lady Michelle Obama's remark, "I know for me, I'm a lawyer because I was bad at [math and science]. All lawyers in the room, you know it's true. We can't add and subtract, so we argue."[4] The legal profession widely accepts the assumption that lawyers cannot do math. This assumption points to lawyers' underlying belief that they very rarely need to engage in mathematical thinking.

Despite the belief that lawyers rarely engage in mathematical thinking, a powerful connection, in fact, exists between legal reasoning and mathematical thinking, as well as lawyers' ability to think mathematically, which refers to numeracy.[5] Studies in numeracy, "the ability to understand and use numbers," have revealed that (1) mathematical thinking impacts a lawyer's ability to conduct substantive legal analysis, and (2) lawyers and "law students are surprisingly good at math."[6] Such findings demonstrate the need to think of mathematical reasoning beyond mere arithmetic, and instead to view it as the

2. "Lawyers bond openly over their distaste for math and accept the same in others. Those who are competent at—or even enjoy—math are seen as an oddity. Only occasionally is the profession's math paralysis criticized or even questioned." Lisa Milot, *Illuminating Innumeracy*, 63 CASE W. RES. L. REV. 769, 771 (2013). *See also* Elie Mystal, *Law Practice: For Rich Kids Who Don't Like Math*, ABOVE THE L. (Mar. 25, 2014, 5:47 PM), https://abovethelaw.com/2014/03/law-practice-for-rich-kids-who-dont-like-math/ [https://perma.cc/5W35-DU73] (stating in its opening that, "I think we've known for a long time that law is a refuge for people afraid of numbers"); Arden Rowell & Jessica Bregant, *Numeracy and Legal Decision Making*, 46 ARIZ. ST. L.J. 192, 193 (2014) ("Law professors, judges, law students, and attorneys themselves routinely assume that lawyers are bad at math. The assumption is so pervasive and casual that it has become a sort of in-group lawyer joke that attorneys tell to each other.").

3. Rice University, *Centennial Lecture Series: Chief Justice John Roberts Speaks at Rice University*, YOUTUBE, at 13:59 (Oct. 26, 2012), https://www.youtube.com/watch?v=UxaFhJ8JVq8 [https://perma.cc/4CAJ-UXQJ].

4. Michelle Obama, Remarks by the First Lady at the National Science Foundation Family-Friendly Policy Rollout (Sept. 26, 2011), https://www.presidency.ucsb.edu/documents/remarks-the-first-lady-the-national-science-foundation-family-friendly-policy-rollout [https://perma.cc/7HES-NTYC].

5. Some scholars have noted that a judge's work is strikingly similar to that of a mathematician. Both the judge and the mathematician reach their conclusions and then hide that fact through well-reasoned writing and creative illumination. Kevin W. Saunders*, Realism, Ratiocination, and Rules*, 46 OKLA. L. REV. 219, 225 (1993).

6. Phil Ciciora, *Research: Poor Math Skills Affect Legal Decision-Making*, ILL. NEWS BUREAU (Apr. 3, 2013), https://news.illinois.edu/view/6367/204846 [https://perma.cc/XF8Y-MVFW]; Rowell & Bregant, *supra* note 2, at 226.

ability to come to final decisions based on evaluating probabilities, risks, or calculations.[7]

Notwithstanding lawyers' aversity to math are ongoing, AI's presence and impact on the legal community continues to expand.[8] For example, companies such as Blue J Legal[9] have developed programs that are able to take basic legal facts and generate a probability of success on the merits along with a legal memorandum explaining the background law.[10] As AI's capability grows, lawyers will have more powerful tools to enhance their ability to perform research and analysis. A lawyer's abilities to research, counsel, and persuade are already enhanced by current AI tools, and lawyers are learning to shift their methodologies to take advantage of these enhancements.[11]

These developments in AI are fueled by mathematical logic.[12] Mathematics fuels AI through disciplines like linear algebra, calculus, and statistics.[13] An adept mathematician possesses strong logical reasoning, understanding the structure of an idea such that they can develop principles that remain true even if the structural components change.[14]

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7. Ciciora, *supra* note *6*. *See also* MIND Research Institute, *Re-imagining Storytelling to Connect Math, Games, and History*, YOUTUBE (July 3, 2018), https://www.youtube.com/watch?v=RXCrsE0PIX0 [https://perma.cc/H9JE-CE22] (arguing that our poor relationship to mathematics is impacting our learning as a whole and proposes storytelling as a solution to increase students' engagement in learning mathematical concepts).

8. Benjamin Alarie, Anthony Niblett & Albert H. Yoon, *How Artificial Intelligence Will Affect the Practice of Law*, 68 UNIV. OF TORONTO L.J. 106, 121–22 (2018); Sean Semmler & Zeeve Rose, *Artificial Intelligence: Application Today and Implications Tomorrow*, 16 DUKE L. TECH. REV. 85, 87–89 (2017).

9. Blue J Legal described its functionality as "the only solution that combines the power of machine learning and AI with expert understanding of legal factors." BLUE J, https://www.bluej.com/ca [https://perma.cc/M7ZQ-574U]. The way Blue J Legal uses machine learning to analyze and predict answers to legal questions in tax and employment law is described in Alarie, Niblett & Yoon, *supra* note 8, at 119. *See also* Benjamin Alarie, Anthony Niblett & Albert H. Yoon, Using Machine Learning to Predict Outcomes in Tax Law 7–8 (2017) (unpublished article), https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2855977 [https://perma.cc/9EKP-W89W].

10. Alarie, Niblett & Yoon*, supra* note 9, at 12. Alexsei is another tool that is described on its website as using AI to provide "high-quality answers to legal questions in memo format." ALEXSEI, https://www.alexsei.com/ [https://perma.cc/BAR2-CSPZ].

11. JOANNA GOODMAN, ROBOTS IN LAW: HOW ARTIFICIAL INTELLIGENCE IS TRANSFORMING LEGAL SERVICES 3 (2016). Richard Susskind discusses the need for lawyers to consider their role to "work in the interests of society" when integrating legal technology into their practice. RICHARD SUSSKIND, TOMORROW'S LAWYERS: AN INTRODUCTION TO YOUR FUTURE 195 (2d ed. 2017).

12. Shafi, *AI & Mathematics*, MEDIUM (Sept. 29, 2020), https://medium.com/swlh/ai-mathematics-699a9ea2a0d6 [https://perma.cc/N5PN-JJXN].

13. *Id.*

14. *See* ELLENBERG, *supra* note 1, at 12–14.

To continue to grow into their shifting role as technology users, lawyers should understand how AI and the underlying mathematics work. The application of AI demonstrates that mathematical models do work in terms of legal research to assess legal outcomes. The connection between AI and law may seem contrary to traditional notions that math is incompatible with law. However, this Article does not suggest that scholars and legal thinkers incorrectly argued that the law is not equivalent to mathematics. Rather, this Article suggests that mathematics can complement legal thinking, just as social science can inform public policy arguments.

As AI comes further into the forefront of lawyering, a lawyer who understands mathematical thinking can use mathematics to inform their legal skills. Understanding mathematics and its principles empowers lawyers to harness AI's power as a legal tool in an ever-advancing world. Lawyers need to understand the bridge between legal reasoning and math. This bridge is demonstrated by the mathematical algorithms used in AI.[15]

This Article will proceed in four parts. In Part I, we describe the traditional narrative that math and the law are incompatible. We explore Oliver Wendell Holmes's metaphor of mathematics, which he used to convey the idea that law cannot be strictly viewed in terms of logic. We discuss philosophers Chaïm Perelman and Lucie Olbrechts-Tyteca's distinctions between mathematical thinking and argumentation, and we assess patent law's view that mathematical innovations are discoveries, not inventions. In Part II, we posit that we need a new mathematical metaphor, one that advances the use of logic in critical thinking about the law, while weaving in policy-based arguments and customary arguments.

Part III presents a primer on math for lawyers. Part IV discusses how to integrate mathematical thinking and legal reasoning and communication. In this part we assess how mathematical logic compares to legal reasoning. We also consider the relationship between mathematics, legal reasoning, and AI. AI can research the law and even analyze how the law applies to a client's facts, in some instances. AI tools are even capable of drafting some contracts and legal memoranda. Since AI is based on mathematical algorithms that drive the legal research and analysis, we argue that lawyers have a greater need to understand the practical realities of the analogs between mathematical reasoning and legal reasoning. In the Recommendations and Conclusion Section, we consider ways logical reasoning can be taught in law schools and used by attorneys and other legal professionals to strengthen their arguments.

15. KATHERINE B. FORREST, WHEN MACHINES CAN BE JUDGE, JURY, AND EXECUTIONER: JUSTICE IN THE AGE OF ARTIFICIAL INTELLIGENCE 12 (2021) (describing algorithms as the "building blocks" of AI).

II. TRADITIONAL NARRATIVES THAT MATH IS INCOMPATIBLE WITH LAW

In his essay, "The Resilience of Law," Joseph Vining poignantly describes his reaction to his father's attempt to quantify the law and economics.[16] His father, Rutledge Vining, created a symbolic notation for determining the statistical effect of legislation on the economy.[17] Vining, while proud of his father, could not bring himself to review his father's work in creating symbols and equations to assess legislation that would affect the economy.[18] He could not do so, he recounts, because "[l]aws might have systematic qualities but law was alive in a way rules that make a system are not."[19] Vining's story illustrates the traditional perception of a strict divide between the law and forms of thinking that use symbolism or mathematics.

Aristotle's writing is the first to have described deductive reasoning, which applies to geometry, but he saw a difference between mathematical reasoning and human rhetoric and argumentation.[20] Aristotle distinguishes, for instance, the wisdom of a person who understands the abstraction of mathematics and geometry, which pertains to the universal, from a person who has the wisdom of human experience, which addresses the particular.[21] Aristotle also differentiates mathematical arguments, which he states do not involve choice, from arguments about people, which bear on their character, as demonstrated by the choices they make.[22] This narrative has continued in the debate between

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16. Joseph Vining, *The Resilience of Law*, *in* LAW AND DEMOCRACY IN THE EMPIRE OF FORCE 151–52 (H.J. Powell & J.B. White eds., 2009).

17. *Id.*

18. *Id.* at 152.

19. *Id.*

20. Brian Larson, *Endogenous and Dangerous*, 22 NEV. L.J. (forthcoming 2022) (manuscript at 4).

21. ARISTOTLE, NICOMACHEAN ETHICS bk. VI, at 98–99 (W.D. Ross trans., Batoche Books ed. 1999) (c. 384 B.C.E.) ("What has been said is confirmed by the fact that while young men become geometricians and mathematicians and wise in matters like these, it is thought that a young man of practical wisdom cannot be found. The cause is that such wisdom is concerned not only with universals but with particulars, which become familiar from experience, but a young man has no experience, for it is length of time that gives experience; indeed one might ask this question too, why a boy may become a mathematician, but not a philosopher or a physicist. It is because the objects of mathematics exist by abstraction, while the first principles of these other subjects come from experience, and because young men have no conviction about the latter but merely use the proper language, while the essence of mathematical objects is plain enough to them?").

22. ARISTOTLE, ART OF RHETORIC bk. III, at 201 (Robert C. Bartlett trans., The Univ. of Chicago Press ed. 2019) ("The narration also ought to bear on [moral] character. This will be the case if we know what fosters [a sense of one's] character. Now, one thing is to make clear the choice involved; and the sort of character one has relates to the sort of choices one makes, and the sort of choice one makes is related to one's end or goal. It is for this reason that mathematical arguments do not involve

Christopher Columbus Langdell and Oliver Wendell Holmes over legal formalism.[23] Further, it has been witnessed in the modern work of Chaïm Perelman and Lucie Olbrechts-Tyteca's *The New Rhetoric*,[24] and it appears in the approach taken in patent law cases to assess the patentability of mathematics. The common theme weaving through these narratives is that human argumentation cannot be reduced to mathematical abstraction or formal logic.

A. The Langdell-Holmes Debate

The relationship between mathematical thinking and legal reasoning appears in the debate between Christopher Columbus Langdell, dean of Harvard Law School, and Oliver Wendell Holmes, Jr., Justice of the Supreme Court and also professor at Harvard during Langdell's tenure as dean.[25] The debate centered on whether law is a science capable, as Langdell asserted, of being uncovered by reading cases and applied through formal logic to a new case.[26] Holmes uses the metaphor of mathematics to express his concern that law should not be reduced to mere logic.[27] This metaphor allows Holmes to argue, as an analog, why law is not a science.

As a legal formalist, Langdell viewed law as a science.[28] He described law as a "legal truth" to be discovered, and he compared the printed books in law

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matters of character, because they do not involve choice either (since they do not have the 'for the sake of which' [or end]), whereas Socratic arguments do, since they speak about such things.").

23. Stephen R. Alton, *Roll Over Langdell, Tell Llewellyn the News: A Brief History of American Legal Education*, 35 OKLA. CITY UNIV. L. REV. 339, 355 (2010).

24. *See generally* CHAÏM PERELMAN & LUCIE OBRECHTS-TYTECA, THE NEW RHETORIC: A TREATISE ON ARGUMENTATION (John Wilkinson & Purcell Weaver trans., Notre Dame Press 1969) (originally published as LA NOUVELLE RHETORIQUE: TRAITE DE L'ARGUMENTATION (Presses Universitaires de France 1958)).

25. Susan Haack, *On Logic in the Law: "Something, But Not All"*, 20 RATIO JURIS 1, 1 (Mar. 2007) (quoting OLIVER WENDELL HOLMES, JR., THE COMMON LAW (1881)).

26. Alton, *supra* note 23, at 346–48. *See also* Haack, *supra* note 25, at 6 (referring to an after-dinner speech by Langdell titled "Teaching Law as a Science," which he gave after Holmes's Path of the Law speech).

27. Alton, *supra* note 23, at 353 (quoting OLIVER WENDELL HOLMES, JR., THE COMMON LAW 1 (1881)).

28. *Id.* at 346. *See also* Todd E. Pettys, *The Analytic Classroom*, 60 BUFF. L. REV. 1255, 1263 (2012) (quoting C.C. LANGDELL, A SELECTION OF CASES ON THE LAW OF CONTRACTS, at vi-vii (1871)); Haack, *supra* note 25, at 3; David S. Romantz, *The Truth About Cats and Dogs: Legal Writing Courses and the Legal Writing Curriculum*, 52 KAN. L. REV. 105, 115 (2003) (noting the case method was a way for students to "objectively and rationally discover the law by isolating and analyzing legal doctrines found in judicial opinions"); Richard Neumann, *Osler, Langdell & the Atelier: Three Tales of Creation in Professional Education*, 10 J. LEGAL COMM. & RHETORIC 151, 166–71 (2013) (describing Langdell's teaching methods).

libraries to the laboratories used in the hard sciences.[29] These comparisons motivated Langdell to secure the place of law as a distinct discipline and graduate course of study.[30] Langdell thought that legal truths were limited in number and could be ascertained by reading cases.[31] Langdell believed these legal truths, as rules of law, could then be applied to a new set of facts, as syllogistic reasoning.[32]

Holmes disagreed with Langdell for reducing the law to formal logic. Generally, Holmes criticized Langdell for ignoring the moral and political forces and public policy that also play a role in judicial decisions and jury verdicts.[33] Holmes has been often quoted as saying:

> The life of the law has not been logic; it has been experience. The seed of every new growth within its sphere has been a felt necessity. The form of continuity has been kept up by reasonings purporting to reduce everything to a logical sequence; but that form is nothing but the evening dress which the newcomer puts on to make itself presentable according to the conventional requirements. The important phenomenon is the man underneath it, not the coat; the justice and reasonableness of the decision, not its consistency with previously held views [. . .]. The law finds its philosophy not in self-consistency, which must always fail so long as it continues to grow, but in history and the nature of human needs. As a branch of anthropology, law is an object of science; the theory of legislation is a scientific study; but the effort to reduce the concrete details of an existing system to the merely logical consequence of simple postulates is always in danger of becoming unscientific, and of leading to a misapprehension of the nature of the problem and the data.[34]

In other words, Holmes sees law as reflecting the human condition in all its messiness, as opposed to mere postulates and theorems. Holmes uses the metaphor of the evening dress to suggest that logical argumentation is a way to

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29. Alton, *supra* note 23, at 347 (quoting Christopher C. Langdell, Dane Professor, Harvard Law Sch., Address to Harvard Law School Association at the "Quarter Millennial" Celebration of Harvard University (Nov. 5, 1886), *in The Harvard Law School*, 3 L.Q. REV. 188, 124 (1887)).

30. *Id.* at 350.

31. *Id.* at 347.

32. *See id.* (noting that according to Langdell, "[i]f one found one of these few legal truths and applied it scientifically to the case at hand, the legal result should be virtually automatic and invariable").

33. *Id.* at 353; Haack, *supra* note 25, at 5.

34. *E.g.*, Haack, *supra* note 25, at 4 (quoting HOLMES, THE COMMON LAW 234 (1880)).

beautify an argument, but it is not the argument itself. He suggests that a sound argument must be something more than correctly reasoned—it must be just.

Holmes elaborated on these points in his address at the dedication of the new hall at the Boston University School of Law in 1897.[35] Titled "The Path of the Law," Holmes spoke of the study of law as the study of a profession.[36] Law is a profession, he said, because people pay lawyers to represent them knowing that the power of the state is behind a judge's decision, and a lawyer must predict "the incidence of that public force through the instrumentality of the courts."[37] In considering how law grows and develops, Holmes took the stance that it would be a "fallacy" to posit that "the only force at work in the development of the law is logic."[38] He admitted that "in the broadest sense that notion would be true," but remarked that:

> The postulate on which we think about the universe is that there is a fixed quantitative relation between every phenomenon and its antecedents and consequents. If there is such a thing as a phenomenon without these fixed quantitative relations, it is a miracle. It is outside the law of cause and effect, and as such transcends our power of thought, or at least is something to or from which we cannot reason. The condition of our thinking about the universe is that it is capable of being thought about rationally, or, in other words, that every part of it is effect and cause in the same sense in which those parts are with which we are most familiar. So in the broadest sense it is true that the law is a logical development, like everything else. The danger of which I speak is not the admission that the principles governing other phenomena also govern the law, but the notion that a given system, ours, for instance, can be worked out like mathematics from some general axioms of conduct.[39]

In this last sentence, Holmes seems to be speaking directly to Langdell's view that the law is governed by a limited number of fundamental legal truths.[40] Here, Holmes uses mathematics as a metaphor for overly simplistic argumentation and analysis.

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35. Oliver Wendell Holmes, Jr., *The Path of the Law*, 10 Harv. L. Rev. 457 (1897), *reprinted in The Path of the Law After 100 Years*, 110 HARV. L. REV. 991 (1997).

36. *Id.* at 991.

37. *Id.*

38. *Id.* at 997.

39. *Id.* at 997–98.

40. *See* Haack, *supra* note 25, at 5.

Holmes also employs the metaphor of mathematics to reject formulaic reasoning and structure of arguments:

> The training of lawyers is a training in logic. The processes of analogy, discrimination, and deduction are those in which they are most at home. The language of judicial decision is mainly the language of logic. And the logical method and form flatter that longing for certainty and for repose which is in every human mind. But certainty generally is illusion, and repose is not the destiny of man. Behind the logical form lies a judgment as to the relative worth and importance of competing legislative grounds, often an inarticulate and unconscious judgment, it is true, and yet the very root and nerve of the whole proceeding. You can give any conclusion a logical form.[41]

Holmes recognizes the value of using logic in both a lawyer's arguments and a judge's decision. He again suggests in this passage that a logical form, as used by a lawyer or a judge, creates a sense of certainty that does not truly reflect the tug-of-war of the underlying policies supporting each party's contentions. Holmes acknowledges that sometimes those competing interests are not even part of our consciousness.

Holmes's metaphor that law is not mathematics focuses on the idea that legal reasoning must be based on practical and sound judgment, considering the public force that Holmes argued underlies our system of law. Certainly, lawyers use traditional forms of logic, but that logic must be supported by a rationale that accounts for social customs and public policy.

B. The View of Philosophy and Rhetoric

Philosophers and rhetoricians have also pondered the relationship between logic, mathematics, and legal reasoning.[42] Philosophers Chaïm Perelman and Lucie Olbrechts-Tyteca undertook the study of the logic of non-formal

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41. Holmes, *supra* note 35, at 998.

42. Chaïm Perelman and Stephen E. Toulmin are examples of modern scholars who have considered this question. *E.g.*, STEPHEN E. TOULMIN, THE USES OF ARGUMENT, at xi (Updated ed., Cambridge Univ. Press 2003). This Article focuses on Perelman and Lucie Olbrechts-Tyteca's *The New Rhetoric*. In his essay, "What the Philosopher May Learn from the Study of Law," Perelman argued that philosophers should look to law, instead of mathematics or science. Francis J. Mootz III, *Right Rhetoric: What Lawyers May Learn From the Study of Rhetoric*, *in* LIBER AMICORUM DE GUY HAARSCHER,QU'EST-CE QUE LA PHILOSOPHIE DU DROIT? (Free Univ. of Brussels 2013) (noting Perelman advocated that philosophers can "look to legal practice and its practical attention to concrete problems that cannot be resolved by applying an algorithm").

arguments.[43] They applied rhetorical theory to the logic of non-formal arguments in their 1958 work, *The New Rhetoric*.[44] The aim of argumentation, they state, is to secure "the adherence of those to whom it is addressed" and, therefore, "it is, in its entirety, relative to the audience to be influenced."[45]

Similar to Holmes's mathematics metaphor, Perelman and Olbrechts-Tyteca use mathematics in *The New Rhetoric* to illustrate how argumentation is different from formal logic. They begin by observing that under mathematicians' influence, logic has come to mean formal logic, which they defined as "the methods of proof used in the mathematical sciences."[46] As such, they posit, any reasoning that does not fall under formal logic "elude[s] reason."[47] The study of methods of proof in logic do not "venture to examine the proofs used in human sciences."[48] While Holmes had asserted that legal argument could not be reduced to mere formal logic, Perelman and Olbrechts-Tyteca now were observing that logic had come to mean purely formal logic, without consideration for other forms of proof.[49] Moreover, logic had become, under the influence of mathematics, symbolic and "no longer related to any rational evidence whatsoever."[50]

The best place to convey the unique characteristics of argumentation, Perelman and Olbrechts-Tyteca note, is to contrast it with the classical concept of demonstration.[51] The modern logician, relying on mathematical reasoning, is "free to fix the symbols and combinations of symbols that may be used."[52] A logician must only choose "symbols and rules in such a way as to avoid doubt and ambiguity."[53] The origin of axioms and rules of deductive reasoning "goes beyond the framework of the formalism in question."[54] Accordingly, the

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43. PERELMAN & OBRECHTS-TYTECA, *supra* note 24, at 142. Perelman undertook the work of developing the theories, and Obrechts-Tyteca created the examples. David A. Frank & Michelle Bolduc, *Lucie Olbrechts-Tyteca's New Rhetoric*, 96 Q. J. OF SPEECH 141, 141–42 (2010).

44. PERLEMAN & OBRECHTS-TYTECA, *supra* note 24, at 142.

45. *Id.* at 19.

46. *Id.* at 2.

47. *Id.*

48. *Id.*

49. In the last century, "[m]odern formal logic became, in this way, the study of the methods of demonstration used in the mathematical sciences." *Id.* at 10. Perelman and Obrechts-Tyteca analyzed arguments from other sources in the "human sciences, law and philosophy" to create their theory of argumentation. *Id.* at 9.

50. *Id.* at 13.

51. *Id.*

52. *Id.*

53. *Id.*

54. *Id.*

meaning and interpretation of the "axiomatic system" is not questioned, and a logician need only be concerned with the "adequacy for the end pursued."[55]

In contrast, Perelman and Olbrechts-Tyteca explain, in argumentation, when people use "discourse to influence the intensity of an audience's adherence to certain theses, it is no longer possible to neglect completely, as irrelevancies, the psychological and social conditions in the absence of which argumentation would be pointless and without result."[56] Argumentation requires "an effective community of minds," and that intellectual community must agree to debate a question.[57] Accordingly, Perelman and Olbrechts-Tyteca's work to define these concepts shows that they view argumentation, and by extension, legal argument, as more complicated and expansive than formal logic or mathematics.

C. An Example from Patent Law

In keeping with the traditional notions advanced by thinkers such as Aristotle, Holmes, and Perelman/Olbrechts-Tyteca, the legal field has been reticent to recognize, if not downright critical of, the idea that mathematics and law are compatible fields. This discord between mathematical and legal thought may be seen in patent law. Patent law is an example of one area of law, among many, that embodies the notion that mathematics and law cannot be compared.

Patent case law continually denies protection to mathematical theories and expression on the basis that math is a discovered truth, not a created invention.[58] In focusing on this dichotomy between discovered truth and created invention, patent law exemplifies the legal community's tendency to ignore the similarities between mathematics and law. Patent law exemplifies this tendency because it ignores the inventive and creative nature of mathematical problem solving. The dichotomy between discovered truth and created

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55. *Id.* at 14.

56. *Id.*

57. *Id.*

58. To understand the underpinnings of the viewpoint that math is a discovered truth as opposed to a created invention, it is useful to have a precise definition for what is an invention is and what is a discovery. An invention, at large, may be viewed as technology that solves a specific problem. *What is an Invention*, IGE IPI, https://www.ige.ch/en/protecting-your-ip/patents/patent-basics/what-is-an-invention [https://perma.cc/N7AR-5Y8W]. A discovery, on the other hand, uncovers an existing truth that was previously unknown. JACQUES HADAMARD, THE PSYCHOLOGY OF INVENTION IN THE MATHEMATICAL FIELD, at xi (1945). The distinction between an invention and discovery then can be thought of using this comparison: "Columbus discovered America: it existed before him; on the contrary, Franklin invented the lighting rod: before him there had never been any lighting rod." *Id.* So, the distinction between invention and discovery is between finding truth and creating something new from existing truths.

invention demonstrates how patent law creates norms that separate the fields of mathematics and law. Patent law's focus on this dichotomy reinforces the differences between the two fields such that it becomes harder to shed light on the commonalities that they share.

First and foremost, patent law protection extends to any created invention, not a discovered truth. Specifically, patent law may protect any person who "invents or discovers any new and useful process, machine, manufacture, or composition of matter, or any new and useful improvement thereof," subject to conditions and requirements.[59] In interpreting this statute, courts have limited patentable subject matter to exclude "the laws of nature, physical phenomena, and abstract ideas."[60] These exclusions extend to mathematics as "an abstract idea . . . and a law of nature."[61] For example, in *Gottschalk v. Benson*, the United States Supreme Court held mathematical algorithms in software are not patentable subject matter because "[p]henomena of nature . . . mental processes, and abstract intellectual concepts" cannot be protected.[62] In other words, mathematics cannot be protected by a patent because it is a naturally occurring truth that is discovered through a mathematician's reasoning. Accordingly, patent law, at its foundation, views mathematics as a truth to be discovered about the world, not a protectable invention.[63]

In viewing mathematics as a truth to be discovered about the world, the United States Supreme Court in *Diamond v. Diehr*, a seminal decision determining mathematics is non-patentable subject matter, demonstrated patent law's attachment to Platonism.[64] Platonism, in the context of patent law, is the idea that mathematics is discovered more than it is created.[65] In refusing to give patent protection to mathematics, patent law relies upon the idea that math is a discovered truth as opposed to an invented creation. The Court in *Diehr*

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59. 35 U.S.C. § 101.

60. *General Information Concerning Patents*, USPTO, (2022), https://www.uspto.gov/patents/basics/general-information-patents [https://perma.cc/2L9B-M77G].

61. *Appendix 1 to the October 2019 Update: Subject Matter Eligibility*, USPTO (Oct. 2019), https://www.uspto.gov/sites/default/files/documents/peg_oct_2019_app1.pdf, [https://perma.cc/7KWB-LGHB].

62. 409 U.S. 63, 67, 73 (1972).

63. *See* Paul B. de Laat, *Patenting Mathematical Algorithms: What's the Harm? A Thought Experiment in Algebra*, 20 INT'L REV. OF L. AND ECON. 187, 188 (2000).

64. *See generally* 450 U.S. 175 (1981).

65. Alec Wilkinson, *What is Mathematics*, THE NEW YORKER (Mar. 2, 2021), https://www.newyorker.com/culture/culture-desk/what-is-mathematics [https://perma.cc/Q897-AP4W]. Rebecca Goldstein describes a mathematical Platonist as someone who "uses the word 'true,' even when applied to mathematical statements, in exactly the same way as we normally use the word, not as a shorthand for 'relative to x' but to represent existing states of affairs." REBECCA GOLDSTEIN, INCOMPLETENESS: THE PROOF AND PARADOX OF KURT GÖDEL 87 (2005).

exemplified this by expanding patent protection to inventions using math, while continuing to exclude math itself as non-patentable subject matter.[66] In *Diehr*, the patent application described the invention as the process of constantly measuring the temperature inside a rubber mold and feeding the measurements into a computer to calculate the appropriate cure time using a mathematical equation.[67] The patent examiner rejected the claims because the claims were based on mathematics and computer programming, non-statutory subject matter.[68]

The Supreme Court granted certiorari on the issue of whether a mathematical equation deserved patent protection.[69] The Court held that a process using mathematical truths may be patentable, but the math itself could not be patented.[70] In its view, math is an abstract principle, a fundamental truth, that cannot be patented because no one person can claim an exclusive right in it.[71] In reasoning that a mathematical formula expresses a law of nature, the *Diehr* Court's rationale embodies the view that math is an inherent truth waiting to be discovered.

This distinction between truth and invention exemplifies the legal field's unwillingness to recognize and understand the inventive critical thinking and reasoning that mathematics requires, a notion tracing back to early scholars such as Plato and Aristotle.[72] But what if this understanding of math as an invented truth was not present in legal thinking? Then scholars might begin to look at the commonalities between the law and mathematics. Removing this continued lens of mathematics as an invented truth allows for the insight that mathematics is often a creative field that requires definition and strong logical reasoning to develop new knowledge.[73] In shifting to this new viewpoint, the similarities between mathematical thinking and reasoning may be further developed. But as long as the legal field, as illustrated by the area of patent law, continues to view mathematics only as a truth to be uncovered, it is

66. *Diehr*, 450 U.S. at 191–93.

67. *Id.* at 177–78.

68. *Id.* at 179–80.

69. *Id.* at 181.

70. *Id.* at 184.

71. *Id.* at 185–90.

72. Wilkinson, *supra* note 65. Patent law seems to approach math in a similar way to Langdell's attempt to create a new science out of law by discovering legal principles that are already in existence in the natural world.

73. *See* ELLENBERG, *supra* note 1, at 12–13. Mathematics is an inherently creative field. Mathematical creativity seeks to uncover patterns, just as all human fields seek to uncover patterns like in the Bible or in counting sheep. *Maths and Storytelling*, BBC RADIO (1999), https://www.bbc.co.uk/programmes/p0054625 [https://perma.cc/4KHN-THNV].

impossible to engage in an interdisciplinary approach that allows mathematical thinking to inform and strengthen legal analysis.

This brief overview of patentable subject matter highlights how patent law, as a modern legal trend, continues traditional thinkers' views that mathematics is incompatible with the law. Patent law exemplifies the legal field's position that mathematics is an inherent truth of the world waiting to be discovered, instead of seeing the possibility of mathematics as a created invention requiring intensive reasoning just as the law does.

III. LAWYERS SHOULD NOT FEAR MATHEMATICS

Despite a storied history of thinkers opposing the idea that mathematics and law are compatible, we posit that mathematical reasoning can inform legal analysis. Holmes's criticism of Langdell hinges upon Langdell's reduction of law to a solely logic-based form with a limited number of legal principles that act as legal truths. In response to Langdell, Holmes rightly viewed law as more than simple logical axioms and postulates. In Holmes's view, law draws upon public policy and other sources to address society's needs. Holmes, like Aristotle and Perelman and Olbrechts-Tyteca, recognized that law is reflected through logical argument and that mathematics is a metaphor for formulaic thinking.

As these thinkers have aptly grasped, mathematics and law are not directly comparable. The two fields are not one and the same. Mathematical reasoning is often confined to the realm of numbers, while legal analysis grapples with a broad array of statutes, case law, and policy shaped by history and power.

However, an interdisciplinary approach to understand the law through the lens of math improves how we, as lawyers, communicate the law to our clients, to judges, to the public at large, and to each other. This approach responds to Holmes's viewpoint that mathematics is a metaphor for formulaic thinking by demonstrating that math is a form of problem solving that uses a myriad of techniques to creatively solve problems. In understanding the two fields' commonalities of deductive reasoning and straightforward communication, lawyers can sharpen their legal analysis and improve how they communicate legal analysis. By comprehending the differences between law and math, lawyers can harness the power of what makes law its own distinct field. For example, by grasping the breadth of material and different avenues legal analysis can take compared to mathematical reasoning, a lawyer can better choose the appropriate method and channel that method more competently. Further, while lawyers might think differently from mathematicians, we should appreciate the rigor of the approach mathematicians take in proofs and linear thinking. Lawyers need to be able to spot logical fallacies and apply a rigorous approach to develop more precise legal reasoning.

As a profession, we as lawyers, have clung too tightly to Holmes's metaphor of mathematics as strictly formulaic thinking at the risk of ignoring the importance of logic. As lawyers, we use straightforward logical thinking in both writing and thinking about the law. While we also rely upon other forms of argument, we should not forget about logic and, in fact, we should pay more attention to it. Logic is an essential element in our reasoning that has been largely lost in the law school curriculum.[74] In defining our discipline and profession, we should not seek to shun math as a metaphor. Instead, we should embrace mathematics as an analog, not as a methodology to apply directly to law. Such a notion requires that we better understand the basis of logic and reasoning to improve the quality of our analysis and argument. In re-imagining Holmes's idea of math as a metaphor for formulaic thinking, the new metaphor should be that math is analogous to law, which requires lawyers to be more precise in their use of legal logic.

This discussion does not contemplate every form of logic, nor does it seek to limit the mathematical field to what is discussed in these pages. These are complex fields that have been studied by great thinkers dating back many centuries. Rather, this Article seeks to open a dialogue about how we can improve legal analysis by looking to other fields' work, specifically by focusing on how mathematics can inform and improve legal reasoning.

In re-imagining Holmes's work and embracing the new metaphor of math as analogous to law, we see that AI demonstrates the viability of this new metaphor. The reality is that AI uses mathematical algorithms to research the law, analyze the law, and write about the law in both contract drafting and memos.[75] Theorists have even contemplated using AI in judicial decision making.[76] AI is moving to the forefront of legal tools, and, with it, the importance of mathematical logic must be reconsidered in relation to legal reasoning. Moreover, lawyers who understand that they must be able to deliver more than the basic skills AI covers, will be the ones to thrive in the practice of law.[77] To that end, while we appreciate Holmes's point that mathematics can

74. Barbara A. Kalinowski, *Logic Ab Initio: A Functional Approach to Improve Law Students' Critical Thinking Skills*, 22 J. LEGAL WRITING INST. 109, 110–11 (2018).

75. Chris Chambers Goodman, *AI/Esq.: Impacts of Artificial Intelligence in Lawyer-Client Relationships*, 72 OKLA. L. REV. 149, 154 (2019).

76. *Id.* at 152.

77. SUSSKIND, *supra* note 11, at 188 (predicting that lawyers will face competition from machines and that "[t]he best and the brightest human professionals will last the longest—those experts who will perform tasks that cannot or should not be replaced by machines"). Susskind suggests that

represent formulaic thinking, it is at tension with the current reality that legal tools rely heavily upon mathematical logic. As AI tools are being developed, lawyers need to understand mathematical logic to fully embrace these tools. But in adopting the new metaphor that embraces mathematical logic, we should not lose sight of Holmes's point that law is a human endeavor with customs, policy, and sound judgment underlying it. Rather, the new metaphor of math as analogous to law emphasizes the importance of a strong foundation in logic, while we as lawyers continue to use all available tools of good lawyering—especially those such as empathy that make us human.[78] The arguments we build of customs, policy, and sound judgment will be that much stronger when we build them upon the strong foundation of logic informed by mathematical reasoning.

IV. A PRIMER ON MATHEMATICS

Defining mathematics is tricky—even "mathematicians [who] know what mathematics is . . . have difficulty saying it."[79] Often, when we think of mathematics, we think of the core subjects taught in primary and secondary school that follow basic arithmetic: algebra, trigonometry, calculus, linear algebra.[80] However, the discipline extends far beyond the climb up the calculus mountain. Life after calculus includes complex and real analysis, linear algebra, combinatorics, geometry, topology, probability and statistics, and mathematical logic.[81] Math goes beyond the merely computational discipline

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lawyers can offer value through their "knowledge, expertise, experience, insight, know-how, and understanding that they can apply in the particular circumstances of their clients' affairs." *Id.* at 189–90. Lawyers will need to capitalize on these skills to succeed in a more technologically advanced profession. *Id.* at 190.

78. Melissa Love Koenig, Jule A. Oseid & Amy Vorenberg, *OK, Google, Will Artificial Intelligence Replace Human Lawyering*, 102 MARQ. L. REV. 1269, 1302–03 (2019).

79. Wilkinson, *supra* note 65; Gunter M. Ziegler & Andreas Loos, "*What is Mathematics?" And Why We Should Ask, Where One Should Experience and Learn That, And How to Teach It*, *in* PROCEEDINGS OF THE 13TH INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION 75 (G. Kaiser ed., 2017), https://link.springer.com/content/pdf/10.1007%2F978-3-319-62597-3_5.pdf [https://perma.cc/NEJ4-ARH3] (discussing the impossibility of defining mathematics as any one thing that often reflects the person defining the subject, more than the subject itself).

80. Underwood Dudley, *What is Mathematics For?*, 57 NOTICES OF THE AM. MATHEMATICAL SOC'Y 608 (May 2010), https://www.ams.org/notices/201005/rtx100500608p.pdf [https://perma.cc/QLA9-2REJ].

81. Dep't of Mathematics, *Life After Calculus*, CORNELL UNIV., https://math.cornell.edu/life-after-calculus [https://perma.cc/EK8Y-UQVM].

that most are familiar with from early education to a field that proves why those computational disciplines reach results that are invariably true.[82]

With all these different topics within the field, it becomes even more challenging to pin down exactly what math is—or is not. Even within the field, there are multiple conflicting definitions of math, with some viewing math as a discovery and others viewing math as a creative invention.[83] In defining mathematics, maybe the clearest place to start is the dictionary.[84] Merriam-Webster defines mathematics as "the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions and of space configurations and their structure, measurement, transformations, and generalizations."[85] That definition, then, points to mathematics as a systematic, linear pursuit to understand numbers and their relationship to each other and the world.

Teaching standards provide insight into what mathematics is, as they outline the key learning objectives and outcomes that students should gain through mathematical education. Math is often taught as a way of understanding the general through the specific, meaning that broad level concepts and theorems are learned through application and problem solving that isolates the concept to be learned.[86] The National Council of Teachers of Mathematics breaks math into two categories of standards: thinking and content.[87] The thinking standards seek to build the necessary skills to

82. RICHARD HAMMACK, BOOK OF PROOF, at viii (2d ed. 2013). For example, a major in mathematics may include study of logic and foundations, analysis, geometry and topology, algebra, combinatorics, and number theory. *See* Dep't of Mathematics, *The Mathematics Major*, YALE UNIV. (2022), https://math.yale.edu/undergraduate/mathematics-major [https://perma.cc/6NRP-2NKV].

83. *See supra* Section II.B. There is an age-old debate about whether math is discovered as a natural truth of the universe or whether it is created as a tool that solves problems using scientific principles. *See also* Dan Falk, *What is Math?*, THE SMITHSONIAN MAG. (Sept. 23, 2020), https://www.smithsonianmag.com/science-nature/what-math-180975882/ [https://perma.cc/3NBC-J6KC].

84. This interpretive method follows the textualist approach, which relies upon dictionary usage as the starting point in an analysis of ambiguous statutory meaning. *See generally* Phillip A. Rubin, *War of the Words: How Courts Can Use Dictionaries In Accordance with Textualist Principles*, 60 DUKE L.J. 167 (2010), https://scholarship.law.duke.edu/cgi/viewcontent.cgi?article=1474&context=dlj [https://perma.cc/LLW5-RXXD].

85. *Mathematics*, MERRIAM-WEBSTER, https://www.merriam-webster.com/dictionary/mathematics?utm_campaign=sd&utm_medium=serp&utm_source=jsonld [https://perma.cc/C2CZ-RYMR].

86. *See generally* NAT'L COUNCIL OF TEACHERS OF MATHEMATICS, EXECUTIVE SUMMARY: PRINCIPLES AND STANDARDS FOR SCHOOL MATHEMATICS 3, https://www.nctm.org/uploadedFiles/Standards_and_Positions/PSSM_ExecutiveSummary.pdf [https://perma.cc/NJ2D-2S3D].

87. *Id.*

mathematically reason, which is comprised of problem solving, communication, reasoning, and connections.[88] Content standards outline specific math topics that students should master to succeed in everyday life.[89] The content standards include estimation, number sense or numeracy, geometry and spatial sense, measurement, statistics and probability, fractions and decimals, and patterns and relationships.[90]

The combination of thinking and content standards provides a basis for students to engage with the material—students who lack an understanding of content may rely too heavily on procedures.[91] The result is that students are less likely to be able to successfully solve similar problems, justify conclusions, apply the concepts in real world applications, explain the solution to others, or deviate from a known procedure to find a faster solution.[92] So, in teaching mathematics, teachers seek to impart both the ability to problem solve through mathematical reasoning and mastery of everyday mathematical concepts to further a student's critical thinking and reasoning skills.

Despite definitional differences within the field, math, at its core, is a type of problem solving.[93] Mathematics is a system of thought where proof lies at the heart.[94] A proof is the process of demonstrating that a theory or concept is, in fact, true through correct logical argument.[95] Mathematics is a way to think about and analyze problems, not rote memorization of theorems and formulas.[96] On the surface, problem solving may seem contradictory to the mathematical teaching standards of content and thinking. However, the standards seek content mastery to further develop mathematical reasoning and concrete skills like balancing a budget to succeed in everyday life. In fact, math can be

88. *Id.* at 4.

89. *Id.* at 3–4.

90. *Id.*

91. *Standards for Mathematical Practice*, COMMON CORE ST. STANDARDS INITIATIVE, http://www.corestandards.org/Math/Practice/ [https://perma.cc/A69V-XCHA].

92. *Id.*

93. D.G. Moursund, *What is Mathematics?*, https://pages.uoregon.edu/moursund/Math/mathematics.htm [https://perma.cc/AFQ5-C3LQ]. George Johnson, *Useful Invention or Absolute Truth: What is Math?*, N.Y. TIMES (Feb. 10, 1998), https://www.nytimes.com/1998/02/10/science/useful-invention-or-absolute-truth-what-is-math.html [https://perma.cc/76TM-6PKD] (discussing conflicting viewpoints of math as a human creation like literature, religion, or banking and as a discovered truth like the Platonian pi in the sky that seek to explain the world around us).

94. Kyle Pratt, Note, What is Math Research?, 3 (expository note), https://faculty.math.illinois.edu/~kpratt4/what_is_math_research.pdf [https://perma.cc/992Y-6ATY].

95. *Id.*

96. EDWARD A. BENDER, MATHEMATICAL METHODS IN ARTIFICIAL INTELLIGENCE, at xviii (Jan. 19, 1999), http://math.ucsd.edu/~ebender/ai_front.pdf [https://perma.cc/7WXK-6WLN].

considered as "a science of not being wrong about things, its techniques and habits hammered out by centuries of hard work and argument."[97] Math is an extension of common sense—"a means by which we can reason about the uncertain, taming if not altogether domesticating it."[98] In other words, a mathematician can use logical deduction to understand that which is uncertain.

The word "proof" likely evokes an image of high school geometry and two column proofs.[99] While a two-column proof is one stylistic approach to proofs, most mathematical thinkers organize their proofs in paragraphs.[100] Proofs are used in almost every area of math including complex analysis, real analysis, linear algebra, and topology.[101] For every area of math, a proof backs up the technique to ensure the technique is true every time it is applied.[102] There are many structures for completing a proof, but three main types are direct proof (proof by construction), proof by contradiction, and proof by induction.[103]

A direct proof is a sequence of statements that begins with statements known to be true and ends with the conclusion seeking to be proved.[104] A direct proof is not the same as deductive reasoning, but rather it is the application of deductive reasoning to a mathematical problem.[105] In a direct proof, the hypotheses is assumed,[106] and the conclusion is achieved through deduction,

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97. ELLENBERG, *supra* note 1, at 2.

98. *Id.* at 425.

99. *Introduction to Proofs*, CK-12, https://www.ck12.org/c/geometry/introduction-to-proofs/lesson/Two-Column-Proofs-BSC-GEOM/ [https://perma.cc/A9GC-J622].

100. Proofs are the mathematical tool to discover new ideas, confirm their accuracy, and to understand these ideas. Proofs stand opposite to the concept of storytelling. In teaching and using proofs, mathematics students learn to intake stories and then challenge those stories, to prove those stories for themselves. Aslanbek Naziev, *Teaching Mathematics: Discovering vs Storytelling*, 9 BRAIN: BROAD RSCH IN A.I. AND NEUROSCIENCE 128, 128 (2018). This method is similar to learning in law school where students are expected to read case law and challenge the resulting holdings that are to be considered as legal truths.

101. *See Math Course Offerings*, ST. NORBERT COLL. (2021), https://www.snc.edu/math/courseofferings.html [https://perma.cc/LP5R-BEPB].

102. *See* HAMMACK, *supra* note 82, at viii.

103. Nissim Lavy, *Types of Mathematical Proofs,* MEDIUM (Jan. 2, 2017), https://medium.com/@nissim.lavy/types-of-proofs-c43ffacc8ada [https://perma.cc/A2L9-JH94].

104. PATRICK KEEF & DAVID GUICHARD, AN INTRODUCTION TO HIGHER MATHEMATICS, 36 (Whitman Coll. Dep't of Mathematics 2021), https://www.whitman.edu/mathematics/higher_math/higher_math.pdf [https://perma.cc/ZSV5-UVTE].

105. *See* Viktor Grigoryan, Math 320 Proof (Sept. 2, 2018), http://web.simmons.edu/~grigorya/320/notes/note2.pdf [https://perma.cc/U3EG-MSEZ].

106. *See* BEVERLY HENDERSON WEST, ELLEN NORMA GRIESBACH, JERRY DUNCAN TAYLOR & LOUISE TODD TAYLOR, THE PRENTICE-HALL ENCYCLOPEDIA OF MATHEMATICS 447–48 (1982).

reasoning directly from the assumption to the desired conclusion.[107] In saying that we assume the hypotheses, we begin with statements that we already know to be true. In a classic two-column geometric proof, these are the "givens."[108] The hypotheses can be made up of assumptions or previously proven statements.[109] An assumption is a definition, an axiom, or a postulate that is the "essential foundation" of a mathematical system.[110] If we had not assumed the truth of these statements at the outset of the problem, we would have to individually prove them through deduction.[111] One example of a direct proof is to prove the theorem "if p, then q." The proof would adopt the following structure:[112]

Proposition: if P, then Q.

Proof. Assume P.

.

.

.

Therefore Q. *QED.*

In moving from the assumption to the conclusion, the mathematician uses deduction to logically prove every statement. As discussed, the proof would typically be written in paragraph format, where each sentence is a logical link to the next.[113]

On the other hand, a proof by contradiction seeks to prove that a statement is true by demonstrating that the opposite of the statement leads to a false conclusion.[114] Proof by contradiction begins by assuming the statement to be

107. John M. Lee, Math 300 Introduction to Mathematical Reasoning: Proof Templates (Fall 2017), https://sites.math.washington.edu/~lee/Courses/300-2017/proof-templates.pdf [https://perma.cc/H9NJ-M9VF].

108. *See* WEST, GRIESBACH, TAYLOR & TAYLOR, *supra* note 106, at 447–48.

109. *Id.*

110. *Id.*

111. *Id.*

112. HAMMACK, *supra* note 82, at 92. *QED* is an abbreviation for the Latin phrase "quod erat demonstradum," meaning "that which was to be demonstrated." It is a notation that is used to demonstrate the end of a proof. Other common notations include an empty or filled in square. Eric Weisstein, *Q.E.D.,* WOLFRAM MATHWORLD (2021), https://mathworld.wolfram.com/QED.html [https://perma.cc/QJW3-5LEM].

113. HAMMACK*, supra* note 82, at 92.

114. HAMMACK, *supra* note 82, at 111–12.

proved is false.[115] Then, through deduction, a proof by contradiction reaches a false result that is not possible.[116] That is, the result either contradicts the original assumption of the proof or something already known to be true.[117] For example, try to prove the theorem: "P." A proof by contradiction begins with assuming "not P." Using direct reasoning, we proceed until we reach a contradiction. Thus, "P" must be true. The proof follows the structure below:[118]

Proposition: P

Proof. Assume not P.

.
.
.

Therefore P. *QED.*

Finally, a proof by induction seeks to prove a set of statements are all true.[119] It is a way to prove that a statement is true for *every* number in a set.[120] One way to visualize the concept underlying induction is with ladders and stairs in the following analogy:

> [I]f you want to show that someone can climb to the nth floor of a fire escape, you need only show that you can climb the ladder up to the fire escape (n=1) and then show that you know how to climb the stairs from any level of the fire escape (n=k) to the next level (n=k+1).[121]

Like a direct proof, a proof by induction uses deductive reasoning to prove each step.[122] The difference between a direct proof and a proof by induction lies in the fact that a proof by induction attempts to prove the truth of a whole set of

115. Katherine Korner & Vicky Neale, *An Introduction to Proof by Contradiction*, NRICH (2011), https://nrich.maths.org/4717 [https://perma.cc/3WJD-93H5]; HAMMACK, *supra* note 82, at 112.

116. Korner & Neale, *supra* note 115.

117. *Id.*

118. HAMMACK, *supra* note 82, at 112.

119. *Id.* at 154.

120. Vicky Neale, *An Introduction to Mathematical Induction*, NRICH (2018), https://nrich.maths.org/4718 [https://perma.cc/72BB-53TC].

121. Christina Sormani, *Proofs by Induction*, CUNY, http://comet.lehman.cuny.edu/sormani/teaching/induction.html [https://perma.cc/8Z42-8VES].

122. HAMMACK, *supra* note 82, at 154–55.

statements, as opposed to a singular statement.[123] The proof follows this structure:

> **Proposition**: The statements $S_1, S_2, S_3, S_4,$ are all true.
>
> *Proof.* (Induction)
>
> (1) Prove that the first statement S_1 is true.
> (2) Given any integer $k \geq 1$, prove that the statement $S_k \Rightarrow S_{k+1}$ is true.
>
> It follows by mathematical induction that every S_n is true. [*QED.*]

[124]

The first step is the base step, which is usually a simple statement that is easy to prove.[125] The second step is the inductive step, which uses direct proof to demonstrate that S_k through S_{k+1} is true by assuming S_k is true.[126] In reaching the conclusion, the proof demonstrates that S_k through S_{k+1} must be true and, therefore, every S_n must also be true.[127]

These three proof styles demonstrate the logical thinking a mathematician uses every day. Each proof is a framework for approaching a problem and using logic to solve the problem in a clear and straightforward manner. There is no room for excess prose or unnecessary big words. In using a proof, a mathematician explains the problem's solution through systematic, clear reasoning.

Ultimately, while mathematics is not easily defined, it can be understood as a system of thought that seeks to solve problems through logical reasoning. The applications of mathematical logic extend far beyond the realm of mathematical number problems. An adept mathematician possesses strong logical reasoning, understanding the structure of an idea such that they can develop principles that remain true even if the structural components change.

V. INTEGRATING MATHEMATICAL THINKING AND LEGAL ANALYSIS

We might think that Abraham Lincoln attributed his rhetorical abilities to the study of orators but, as mathematician Jordan Ellenberg notes, Lincoln gave credit to Euclid:

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123. HAMMACK, *supra* note 82, at 154–55.
124. *Id.* at 156.
125. *Id.* at 154.
126. *Id.*
127. *Id.*

> In the course of my law-reading I constantly came upon the word 'demonstrate.' I thought, at first, that I understood its meaning, but soon became satisfied that I did not . . . . At last I said, 'Lincoln, you can never make a lawyer if you do not understand what demonstrate means'; and I left my situation in Springfield, went home to my father's house, and stayed there till I could give any propositions in the six books of Euclid at sight.[128]

In studying Euclid, Lincoln developed his integrity because, Ellenberg asserts, Lincoln believed that a person should be able to back up what they say—to be able to demonstrate they are right.[129]

As Lincoln knew, while mathematics and law are distinct fields, understanding the similarities and differences between the two fields strengthens a lawyer's ability to soundly reason and advocate effectively. Legal reasoning and mathematical proofs can follow a parallel structure—for example, legal reasoning uses deductive reasoning, counterargument, and analogy, which correspond to direct proof, contradiction, and induction in math. This type of reasoning underlies AI, which lawyers use in day-to-day tasks like research on a platform like Westlaw or LexisNexis.[130] While lawyers use these platforms every day (and often many times a day), not as many understand the underlying algorithms driving the platforms. In understanding mathematical reasoning, lawyers can better use these tools.

In this Part, we describe how common tools of logic—deductive reasoning, analogy, and counterargument—resemble mathematical forms. We then describe the mathematical logic that underlies AI. Our goal is to bridge the gap perceived between mathematical logic and legal reasoning. We recognize that the two forms of logic are not exactly the same. As scholars such as Stephen Toulmin have recognized, practical logic cannot be perfectly equated with formal logic.[131] And yet, mathematical algorithms are being used to build tools that can analyze the law. We argue that this paradox is worth considering

128. Jordan Ellenberg, *What Honest Abe Learned From Geometry*, WALL ST. J. (May 22, 2021) (emphasis omitted) (quoting Lincoln's response in 1865 to Reverend J.P. Gulliver, who asked him to artGEOMETRY OF INFORMATION, BIOLOGY, STRATEGY, DEMOCRACY, AND EVERYTHING ELSE (2021)), https://www.wsj.com/articles/what-honest-abe-learned-from-geometry-11621656062 [https://perma.cc/LJ6Q-CQ7S].

129. *Id.* (stating that "[i]n Euclid, Lincoln found a language in which it's very hard to dissimulate, cheat or dodge the question. Geometry is a form of honesty."). Ellenberg recounts Lincoln's friend Henry Clay Whitney, who said of Lincoln: "It was morally impossible for Lincoln to argue dishonestly; he could no more do it than he could steal; it was the same thing to him in essence, to despoil a man of his property by larceny, or by illogical or flagitious reasoning." *Id.*

130. *See* discussion *infra* Section V.C.

131. *See generally* TOULMIN, *supra* note 42.

further as lawyers increasingly use AI tools in the practice of law and seek to grow the practice of law even in the face of potential competition from these tools.

A. Mathematics and Legal Reasoning Run Parallel

Law is a discipline that can be compared to mathematics, science, philosophy, and rhetoric, but, of course, law is a unique discipline. No discipline perfectly overlaps, certainly, but understanding and appreciating the similarities, and potential differences, between mathematical thinking and legal reasoning can aid a lawyer in becoming a stronger thinker and advocate. Legal reasoning and mathematical proofs are parallel in structure. Legal reasoning uses deductive reasoning, counterargument, and analogy, which correspond to direct proof, contradiction, and induction in math.

Holmes correctly asserted that law and legal argument include more than logical or mathematical assertions[132] and, yet, by understanding the similar structure of mathematical proofs and legal reasoning, a lawyer can articulate arguments in a more cogent way to better communicate a legal position to others.[133] A more clearly articulated position is naturally more persuasive. Such a position accounts for the strengths and weaknesses of the advocate's argument, as well as the other side's best positions, which allows a judge or jury to get to the bottom of each sides' arguments and see a path to a favorable conclusion. These same benefits apply when a lawyer needs to articulate arguments to a client to assess the next strategic steps or in arguing a position to opposing counsel in settlement negotiations.

This section demonstrates how deductive reasoning, analogy, and counterargument in legal reasoning should be structured to effectively communicate legal arguments. These forms parallel the natural structure of mathematical proof. We recognize that the form of legal reasoning cannot be exactly the same as a mathematical proof because the facts of a case are not indisputably true, and laws change from one jurisdiction to another, or over time, as policy rationales or customs shift the laws. Our point is that the communication of those positions can be aided by structuring the argument to mimic mathematical proof, and we offer some examples from the United States Supreme Court's decision in *Carpenter v. United States*, which show how these forms are used in legal reasoning.

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132. Holmes, *supra* note 35, at 998.

133. Richard K. Neumann, Jr.'s textbook on legal analysis and writing convincingly connects legal reasoning to mathematical proof. Neumann masterfully uses the term proof to exemplify how to build legal analysis and reflect that analysis in writing. *See* RICHARD K. NEUMANN, JR., LEGAL REASONING & LEGAL WRITING: STRUCTURE, STRATEGY, & STYLE (Wolters Kluwer 6th ed. 2009).

1. Deductive Reasoning

Parallels exist between the deductive reasoning used in legal analysis and a direct mathematical proof. While the rules of law differ from the assumptions of mathematical proof, these two formats conduct analysis in the same way. A direct proof simply uses deductive reasoning. The key difference is the content to which the deductive reasoning is being applied.

In deductive reasoning, legal analysis starts with a thesis stating a conclusion.[134] A legal writer must synthesize the various rules of law that govern the issue.[135] Commonly, this process results in what is often called a rule synthesis paragraph,[136] and that paragraph typically precedes the analysis of how those rules apply to the facts. Rules can be stated expressly in the precedent or legal authorities being cited, or they can be implied from the analysis of a precedent decision.[137] The legal standards are similar to the major premise in a syllogism, although the parties might argue about what the rules are, or should be, and the rules can vary from one jurisdiction to another.[138] To that extent, rules of law differ from the major premise in mathematical proofs, but they are conveyed in the same way for the purpose of clear communication.

The conclusion thesis states the overall conclusion that results from the application of the various rules stated in the synthesis to the particular facts of the case.[139] This conclusion is the probative point the lawyer is making in the argument, or the court's overall decision, and would be like the "q" in the mathematical proof. The lawyer or judge must shore up the facts that support the conclusion, but then address the facts that do not support the conclusion in addressing counterarguments.[140] The deduction is more effective the more particularly the facts are stated with concrete details. An advocate does not

134. Deductive reasoning is also referred to as rule-based reasoning. JOAN M. ROCKLIN, ROBERT B. ROCKLIN, CHRISTINE COUGHLIN & SANDY PATRICK, AN ADVOCATE PERSUADES 131 (2016) (outlining the structure starting with a conclusion).

135. *E.g.*, DEBORAH A. SCHMEDEMANN & CHRISTINA L. KUNZ, SYNTHESIS: LEGAL READING, REASONING, & WRITING 44–45 (3d ed. 2007) (describing the process). Mary Beth Beazley describes the importance of finding the "key terms" in the rule, which she calls the "the phrases-that-pay." MARY BETH BEAZLEY, A PRACTICAL GUIDE TO APPELLATE ADVOCACY, 68–71 (4th ed. 2014).

136. NEUMANN, *supra* note 133, at 155–56. *See generally* Michael D. Murray, *Rule Synthesis and Explanatory Synthesis: A Socratic Dialogue Between IREAC And TREAT*, 8 LEGAL COMMC'N & RHETORIC: JAWLD, 217, 223 (2011).

137. RICHARD K. NEUMANN, JR. & SHEILA SIMON, LEGAL WRITING 33 (2008).

138. BEAZLEY, *supra* note 135, at 93.

139. ROCKLIN, ROCKLIN, COUGHLIN & PATRICK, *supra* note 134, at 131–32.

140. *Id.* at 132. Wilson Huhn notes that a case may present more than one syllogism, or even many syllogisms. Wilson Huhn, *The Use and Limits of Syllogistic Reasoning in Briefing Cases*, 42 SANTA CLARA L. REV. 813, 835–36 n.99 (discussing Aldisert's polysyllogistic model of legal reasoning).

need to restate the rules but should apply them directly to the facts of a case. The conclusion is then restated at the end.

Take, for example, *Carpenter v. United States*, where the Court advanced its own argument through a rule statement in support of its conclusion.[141] In Section IIIA of the Court's opinion, the following rule statement advanced the Court's conclusion that a warrant is required for the government to obtain cell phone tower records:

> A person does not surrender all Fourth Amendment protection by venturing into the public sphere. To the contrary, "what [one] seeks to preserve as private, even in an area accessible to the public, may be constitutionally protected." A majority of this Court has already recognized that individuals have a reasonable expectation of privacy in the whole of their physical movements. Prior to the digital age, law enforcement might have pursued a suspect for a brief stretch, but doing so "for any extended period of time was difficult and costly and therefore rarely undertaken." For that reason, "society's expectation has been that law enforcement agents and others would not—and indeed, in the main, simply could not—secretly monitor and catalogue every single movement of an individual's car for a very long period."[142]

These sentences represent fundamental standards of constitutional criminal investigation. The Court set out these standards in the first paragraph of Section A to establish the ground rules for the discussion and application of those standards. In these sentences, we can see a general standard of expectation of privacy, supported by sub-rules, or supporting legal standards, in the remaining sentences that further describe or delineate the main standard in the first sentence.

In the beginning of the next paragraph, the Court applied these standards to the facts in Carpenter's case:

> Allowing government access to cell-site records contravenes that expectation. Although such records are generated for commercial purposes, that distinction does not negate Carpenter's anticipation of privacy in his physical location. Mapping a cell phone's location over the course of 127 days provides an all-encompassing record of the holder's whereabouts.[143]

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141. 138 S. Ct. 2206, 2217 (2018).

142. *Id.* (internal citations omitted).

143. *Id.*

In the first sentence, the Court articulated its conclusion that cell phone tower records carry an expectation of privacy that is violated when the government accesses the records without a warrant to determine a person's physical location.[144] Notably, the Court articulated a broad conclusion in the first sentence that affirmatively applies to future cases and then stated the specific application to Carpenter's case in the next sentence.[145] The Court's reasoning applied the sub-rules from the previous paragraph where it asserted that mapping a person's location over an extended period of time creates a record that is all-encompassing.[146]

In a direct mathematical proof, the hypothesis and conclusion are known to be true because each part of the proof is true. We can think of a direct proof as an old-school two column geometry proof, where each line is assumed to be true and has been proven in a previous theorem. In legal reasoning, the legal standards articulated are supported by citations, although each standard could potentially be changed over time as the precedent unfolds. The facts in an appellate decision are considered true for the purposes of the appeal because they are limited to those facts provided in the record, which "must be accepted as true."[147] At the trial level, each side is allowed to proffer evidence that supports a version of the factual story.[148] Ultimately, however, a jury must decide the "truth" of the facts and apply the legal standards agreed upon by the parties and the court in the jury instructions.[149] The jury instructions act as a major premise, and the jury must fill in the factual assertions.

As such, the legal standards or rules articulated first are the givens that create the field of analysis when a court (or jury) considers the facts of the case. In a trial or traditional court of appeals case, those givens have been asserted in a prior appellate decision. In a case before the United States Supreme Court or a state supreme court, the court can consider the logical extension or modification of the law based on a new set of facts. Even in those cases, as exemplified in the *Carpenter* case, the court must first consider the "given" set of rules that were laid out in previous cases by the majority of the court.

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144. *Id.*

145. *Id.*

146. *Id.*

147. *See* Adam N. Steinman, *The Rise and Fall of Plausibility Pleading?*, 69 VAND. L. REV. 333, 358 (2016).

148. *See Trial Court,* BALLOTPEDIA, https://ballotpedia.org/Trial_court [https://perma.cc/MQ98-Y5TW].

149. *Courts and Legal Procedure: The Role of Juries*, A.B.A. (Sept. 09, 2019), https://www.americanbar.org/groups/public_education/resources/law_related_education_network/how_courts_work/jury_role/. [https://perma.cc/X46E-A8YE].

As noted in Part IIA of this Article, both ancient and modern philosophers and rhetoricians have expressed concern over the possibility of conflating mathematical reasoning with human argumentation. Philosopher Stephen Toulmin's work on the logic of human argumentation is worth noting as we discuss deductive reasoning because Toulmin raises concerns about the use of syllogisms. Stephen Toulmin argues that syllogisms are insufficient for practical argumentation, including legal argumentation.[150] Toulmin viewed syllogisms as overly simple logic forms, and he was also concerned that formal analytics is too rigid for the nature of human argumentation.[151] To address these concerns, Toulmin identified additional components of practical arguments.[152] He argued that a practical argument uses additional structural components not found in a syllogism as used in formal analytics, and that these additional components correlate more practically with human argumentation.[153] Notably, he identified the warrant, which he defined as a step to explain how to get from the data to the conclusion.[154] The warrant, and Toulmin's other parts of an argument, make sense in the context of human argumentation and, in particular, legal argumentation. We do not disagree with Toulmin. Our purpose here, however, is two-fold: 1) to remind ourselves that parallels exist in the structure of mathematical reasoning and legal deduction, and 2) to consider how those similarities are being used to create algorithms that have real-world applications.

Chaïm Perelman also rightly expressed a distinction between the use of mathematics to demonstrate the truth of a statement, versus the adherence to an argument that an advocate seeks in a human argument.[155] We similarly do not seek to clarify or oppose Perelman's position that mathematics and legal reasoning are fundamentally different—mathematics seeks to demonstrate a truth while legal reasoning seeks to argue a point.[156] Rather, we recognize

150. TOULMIN, *supra* note 42, at 100 ("[W]e shall find that the apparently innocent forms used in syllogistic arguments turn out to have a hidden complexity.").

151. *Id.* at 117 ("If the purpose of an argument is to establish conclusions about which we are not entirely confident by relating them back to other information about which we have greater assurance, it begins to be a little doubtful whether any genuine, practical argument could *ever* be properly analytic.") (emphasis in original).

152. *See id.* at 87–134 (identifying and describing an argument's various components).

153. *Id.*

154. *Id.* at 90–91.

155. CHAÏM PERELMAN, THE REALM OF RHETORIC 48 (1982) (translation of original published as L'EMPIRE RHÉTORIQUE: RHÉTORIQUE ET ARGUMENTATION) ("There is a tendency among formalistic logicians to reduce all deductive reasoning to a demonstration which would be correct if the operations agreed with a pre-established scheme and incorrect if they did not.").

156. *Id.*

deductive reasoning's basic organizational structures have been adopted—and adapted—to suit the needs of advocates.

2. Counterargument

In legal reasoning, a lawyer must account for counterarguments.[157] One of the challenges of good lawyering is to identify the other side's best arguments, to acknowledge those arguments to oneself—considering them and assessing how best to respond to them—and then to craft the response.[158] This process can invoke fear in the heart of an advocate facing the other side's best arguments, but it also provides a lawyer with a uniquely satisfying challenge. An attorney must be able to convey the other side's best arguments to the client, so that together they can most effectively consider the next steps in a case and assess the costs and benefits of strategic decisions. Properly assessing counterarguments helps an attorney and a judge to understand the greatest point of tension in a controversy.[159]

Counterarguments in legal reasoning are akin to mathematical contradiction. A legal counterargument might be premised on a different interpretation of the relevance of certain facts to the argument, or how best to apply the holding or legal standards of a case. A counterargument may be based on deductive reasoning or analogy. Like a proof by contradiction, an advocate must start by identifying the counterargument. An advocate must also fill in the reasoning that could potentially support the counterargument. The argument must be built up, so it can be knocked down, either by showing that the legal standards do not apply in that way to the facts of the case, or that a case precedent can be distinguished. As such, an advocate can demonstrate that an opposite position creates a result that conflicts with the original assumption, the law.

To illustrate, in *Carpenter*, the government asserted that the third-party doctrine applied to the cell phone record information.[160] Under the third-party doctrine, which is an exception to the warrant requirement, a warrant is not needed to access information that is used commercially and, therefore, is already in the public sphere.[161] The government pointed to the Court's prior

157. KRISTIN KONRAD ROBBINS-TISCIONE, RHETORIC FOR LEGAL WRITERS: THE THEORY AND PRACTICE OF ANALYSIS AND PERSUASION 196 (2009) (identifying two forms of counterargument: denial of the allegation and a superseding argument that negates liability even if the assertion is proved).

158. *Id.* at 197–201 (describing how to factor in a counterargument persuasively). Robbins-Tiscione discusses the importance of "diminish[ing] the impact" of a counterargument. *Id.* at 200.

159. *Id.* at 201 (an advocate needs to present a counterargument and response to highlight which argument is "most fair").

160. *Carpenter*, 138 S. Ct. at 2219.

161. *Id.*

decisions in *Smith v. Maryland* and *United States v. Miller* to support its argument, as noted by the Court:

> The Government's primary contention to the contrary is that the third-party doctrine governs this case. In its view, cell-site records are fair game because they are "business records" created and maintained by the wireless carriers. The Government (along with Justice Kennedy) recognizes that this case features new technology, but asserts that the legal question nonetheless turns on a garden-variety request for information from a third-party witness.[162]

Here, the Court identified the counterargument, as well as the government's reasoning in support of the counterargument. But the Court found that line of reasoning unpersuasive:

> The Government's position fails to contend with the seismic shifts in digital technology that made possible the tracking of not only Carpenter's location but also everyone else's, not for a short period but for years and years. Sprint Corporation and its competitors are not your typical witnesses. Unlike the nosy neighbor who keeps an eye on comings and goings, they are ever alert, and their memory is nearly infallible. There is a world of difference between the limited types of personal information addressed in *Smith* and *Miller* and the exhaustive chronicle of location information casually collected by wireless carriers today. The Government thus is not asking for a straightforward application of the third-party doctrine, but instead a significant extension of it to a distinct category of information.[163]

Counterarguments allow the parties and the court to fully develop the strengths and weaknesses of the arguments to find the locus or central point of controversy.

3. Analogy

An important form of argument in legal reasoning is the analogy.[164] An analogy allows an advocate or judge to compare the current case to previous

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162. *Id.* (internal citations omitted).

163. *Id.*

164. Jacob Carpenter, *Persuading with Precedent: Understanding and Improving Analogies in Legal Argument*, 44 CAP. UNIV. L. REV. 461, 465 (2016). Ruggero Aldisert asserts that he finds it convenient to classify analogy as a form of inductive reasoning but notes that not all scholars of logic would agree. Ruggero Aldisert, *Logic in Forensic Science, in* FORENSIC SCIENCE AND LAW: INVESTIGATIVE APPLICATIONS IN CRIMINAL, CIVIL AND FAMILY JUSTICE 11, 23 (Cyril H. Wecht & John T. Rago eds., 2006).

case law.[165] The best way to do that is to consider the facts, holding, and reasoning of the precedent and how those components line up with the present case.[166] An analogy considers the similarities and differences between the precedent and the current case and how that comparison furthers the conclusion being driven.[167] Analogies can be used to further the argument being presented in a case or to help a court understand three-dimensionally how the rules have worked in prior precedent.[168]

For example, in the *Carpenter* decision, the Court had to address a significant counterargument based on existing case precedent, and it handled the counterargument by responding with an analogy.[169] An exception to the warrant requirement exists for commercial records.[170] The Court acknowledged that exception where it used the phrase "[a]lthough such records are generated for commercial purposes."[171] The Court prefaced that sentence with the word "although," which is a signpost for the introduction of a counterargument.[172] The Court went on to respond to the counterargument, noting that such a "distinction does not negate Carpenter's anticipation of privacy in his physical location."[173] The Court found more persuasive that the long-term mapping of Carpenter's physical location provides an "all-encompassing record" of his physical location.[174] The Court supported its position with an analogy to GPS monitoring and information, a fact pattern it had already addressed in *United States v. Jones*:

> As with GPS information, the time-stamped data provides an intimate window into a person's life, revealing not only his particular movements, but through them his "familial, political, professional, religious, and sexual associations." These location records "hold for many Americans the 'privacies of life.'" And like GPS monitoring, cell phone tracking is remarkably easy, cheap, and efficient compared to traditional

165. Carpenter, *supra* note 164, at 486–87.

166. ROCKLIN, ROCKLIN, COUGHLIN & PATRICK, *supra* note 134, at 127.

167. *Id. See also* RUGGERO J. ALDISERT, LOGIC FOR LAWYERS: A GUIDE TO CLEAR LEGAL THINKING 51–52 (3d ed. 1997) (emphasizing the importance of the relevancy of the analogy to the overall argument).

168. MICHAEL D. MURRAY & CHRISTY DESANCTIS, OBJECTIVE LEGAL WRITING & ANALYSIS 10 (2006). *See also* Brian Larson, *Law's Enterprise: Argumentation Schemes & Legal Analogy*, 87 UNIV. CIN. L. REV. 663, 672 (2019) (quoting the work of Neil MacCormick) (describing analogy as most often used for classification and evaluation).

169. *Carpenter*, 138 S. Ct. at 2217.

170. *Id.*

171. *Id. See supra* Section V.A.1.

172. *Carpenter*, 138 S. Ct. at 2217.

173. *Id.*

174. *Id.*

> investigative tools. With just the click of a button, the Government can access each carrier's deep repository of historical location information at practically no expense.[175]

Here, the Court was able to demonstrate that factually the information gleaned from cell phone tower records would provide the same kind of intimate details about a person's life and personal associations as can be determined from GPS data.[176] Because the Court had already ruled that GPS data was protected by meeting the standard for protecting the privacies of life, the Court extended that rationale to cell phone tower data.[177]

The Court then moved on to explain that the facts in the *Carpenter* case are an even better example than *Jones* of the application of the principle of protecting a person's expectation of privacy:

> In fact, historical cell-site records present even greater privacy concerns than the GPS monitoring of a vehicle we considered in *Jones*. Unlike the bugged container in *Knotts* or the car in *Jones*, a cell phone—almost a "feature of human anatomy,"—tracks nearly exactly the movements of its owner. While individuals regularly leave their vehicles, they compulsively carry cell phones with them all the time. A cell phone faithfully follows its owner beyond public thoroughfares and into private residences, doctor's offices, political headquarters, and other potentially revealing locales. Accordingly, when the Government tracks the location of a cell phone it achieves near perfect surveillance, as if it had attached an ankle monitor to the phone's user.[178]

Just as in induction, the Court here was showing that both cases are true; indeed, Carpenter's case perhaps even better illustrates the principle than *Jones*. As such, the Court in *Carpenter* could demonstrate that the cell phone, in addition to (and even more so) than a car, leaves traces of our private lives through location information.[179] These examples in *Carpenter* and *Jones* serve to validate and define by example the generic "black letter law" of a person's expectation of privacy.

Crafting a perceptive and persuasive analogy is an especially challenging endeavor. For one, an advocate must choose what is legally relevant to

175. *Id.* at 2217–18 (internal citations omitted).

176. *Id.*

177. *Id.*

178. *Id.* at 2218 (internal citations omitted).

179. *Id.*

compare.[180] Perelman and Olbrechts-Tyteca argue that analogies do not share the same characteristics with mathematical proportion.[181] In this, they disagree with other scholars, who have found analogies to show a "resemblance of relationship" with "the purest type of analogy" being in "mathematical proportion."[182] Perelman and Olbrechts-Tyteca have a more nuanced view of analogies. They focus on the "difference between the relations involved" in the analogy.[183] They call the terms that buttress the argument the *phoros*.[184] They call the terms that relate to the conclusion the *theme*.[185] The phoros and the theme should have an asymmetrical relation and they should also belong to different spheres.[186] For that reason, they seem to conclude, an analogy cannot be mathematical.[187] This example from *The New Rhetoric* is provided here to emphasize the importance of considering the terms being compared. While a lawyer cannot compare two cases with exact mathematical precision, because no facts line up just right, we can aim to set up an argument to emphasize the right similarities and differences.

While analogies are common place in legal thinking and writing, Robbins-Tiscione identifies three main problems with analogies.[188] One problem occurs when critical information is left out of the cases being compared.[189] Another problem can be the reliance on a case with an adverse outcome.[190] Finally, analogies that address multiple factors must address "relevant or unfavorable

180. ROBBINS-TISCIONE, *supra* note 157, at 173 ("The success of an analogy depends on how strong the reader perceives the factual similarities between the two cases and whether any differences strike the reader as even more significant.").

181. PERELMAN & OLBRECHTS-TYTECA, *supra* note 24, at 372–73.

182. *Id.* at 372 (emphasis omitted) (translating PAUL GRENET, LES ORIGINS DE L'ANALOGIE PHILOSPHIQUE DANS LES DIALOGUES DE PLATON 10 (1948)).

183. *Id.* at 373. The New Rhetoric gives an example of an analogy used by Aristotle: "For as the eyes of bats are to the blaze of day, so is the reason in our soul to the things which are by nature most evident of all." *Id.*(quoting ARISTOTLE, METAPHYSICS bk. II, at Part I (W.D. Ross trans., Oxford Press) (c. 350 B.C.E.))

184. *Id.* In the preceding analogy given by Aristotle, the eyes and bats and the blaze of day together provide the terms for the phoros. *Id.*

185. *Id.* In Aristotle's analogy, the theme is comprised of the terms reason in the soul and obviousness. *Id.*

186. *Id.*

187. *See id.*

188. ROBBINS-TISCIONE, *supra* note 157, at 174–78.

189. *Id.* at 174–75 (stating that the court's reasoning, facts, and holding need to be compared for a complete analogy).

190. *Id.* at 174 (an advocate needs to acknowledge an adverse outcome so the reader can evaluate the importance of the case).

factors" so the reader can see the interplay of the factors.[191] In short, a successful analogy relies on the strength of the details being compared.[192]

As we have demonstrated here, lawyers use deductive reasoning, analogy, and counterargument routinely in their arguments. We can see the parallels to mathematical reasoning. While scholars have argued they are not the same forms of reasoning, and that legal argumentation is more complicated that mathematical forms, they do share similarities. It is time to leave the notion that mathematics cannot inform law. We can recognize that they are distinct disciplines, each bringing value to the scholarly table. But we should recognize the basic similarities. Recognizing those similarities, and actively embracing knowledge of mathematics, can sharpen our reasoning and allow us to see patterns of thought. Moreover, as we discuss in the next section, mathematics underlies the very tools of AI that lawyers are already using. That seems like a paradox, and we encourage both lawyers and mathematicians to engage in discussion to tease out an answer to that paradox. Doing so will not only further our understanding of practical legal logic, but it will help lawyers and developers of technological products to see the possibilities in collaboration.

B. The Mathematics in AI

When we think of AI, in 2022, the time of this Article's publication, we are most likely to think of services like Alexa or Siri, which can use voice commands to retrieve information or do simple tasks in the home or on a person's computer. But what is AI? What mathematics underlies AI? And how is it currently used in the law? Members of the legal profession might know that platforms like Westlaw and LexisNexis use AI. In using the platforms, we are not conscious of the underlying algorithms that drive the functionality, but we know that we can do more complex tasks more easily.

Simply put, mathematics and AI are two branches of the same tree. In fact, mathematics fuels AI through disciplines such as linear algebra, calculus, and statistics.[193] At times, the fields collapse into one another as if they are the same field, and at other times the fields feel wholly distinct with their own language, goals, and methodologies. This section provides a brief overview of the relationship between mathematics and AI to demonstrate the connections the two fields share.

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191. *Id*. at 176–78. Robbins-Tiscione notes that an attorney cannot "cherry-pick" relevant factors. *Id.* at 178.

192. RUGGERO ALDISERT, WINNING ON APPEAL: BETTER BRIEFS AND ORAL ARGUMENT 262 (2d ed. 2003) (An analogy depends on the "*quality of resemblances* between things.").

193. Shafi, *supra* note 12.

AI is the convergence of computers, math, and data.[194] AI, like other scientific fields, requires mathematical thinking and, in fact, mathematics serves as the major designing tool that shapes the field.[195] More explicitly, AI is "a subcategory of computer science that relies on mathematics, including calculus, probability and statistics, linear programming, and other numerical techniques."[196] But to boil it down to its most simple form, AI is really just mathematics conducted on an enormous scale.[197]

Mathematical logic underlies AI software and expert systems.[198] AI relies upon logic programming to develop expert systems.[199] For example, logical AI programs will use sentences of mathematical logical language to represent a problem through given facts about the world and a scenario and the desired goal for the program to reach.[200] "In order to make decisions, a logical AI program infers that certain actions will allow it to achieve a goal."[201] Through this process, logic programs rely upon mathematical logic including proofs in group theory and other parts of mathematics.[202] From this standpoint then, AI can be viewed as "mechanized mathematical logic."[203]

Three key mathematical concepts that underly machine learning, and thus AI, are data, models, and learning.[204] Data is the numerical input for the program to consider or the numerical output for the program to generate, often described in the form of a vector.[205] A model generates data by predicting what

194. Joe Sasson, *Demystifying Artificial Intelligence,* MEDIUM (June 18, 2021), https://medium.com/geekculture/demystifying-artificial-intelligence-bdd9a117d4a6 [https://perma.cc/95QC-MVQX].

195. BENDER *supra* note 96, at xv.

196. Rachel L. Schwein, *Patentability and Inventorship of AI Generated Inventions,* 60 WASH L.J. 561, 566 (2020), https://www.washburnlaw.edu/publications/wlj/issues/60-3.html [https://perma.cc/9EAF-Q2ZA].

197. Shafi, *supra* note 12; Oren Etzioni, *Deep Learning Isn't a Dangerous Magic Genie: It's Just Math*, WIRED (June 15, 2016, 7:00 AM), https://www.wired.com/2016/06/deep-learning-isnt-dangerous-magic-genie-just-math/ [https://perma.cc/A7EL-NAUL].

198. Frederick Hoffman *Introduction and History, in* 55 PROCEEDINGS OF SYMPOSIA IN APPLIED MATHEMATICS: MATHEMATICAL ASPECTS OF ARTIFICIAL INTELLIGENCE 1, 3 (Frederick Hoffman ed., 1996).

199. *Id.*

200. Schwein, *supra* note 196, at 568 n.60.

201. *Id.*

202. Hoffman, *supra* note 198, at 3–4.

203. *Id.* at 3.

204. MARC PETER DEISENROTH, A. ALDO FAISAL & CHENG SOON ONG, MATHEMATICS FOR MACHINE LEARNING 12 (Cambridge Univ. Press 2020), https://mml-book.github.io/book/mml-book.pdf. [https://perma.cc/75EG-JEDP].

205. *Id*.

would happen in the real world without performing real world experiments.[206] Finally, learning is the process in which the model is trained to use the available data to optimize model parameters so that the model can successfully perform on unseen data.[207] These concepts follow the generalized goals of applied mathematics: to apply mathematics to real-world problems to explain observed phenomena and predict new phenomena.[208]

While mathematics and AI are distinct fields, the two fields share the same logical basis to approach real world problems. Each field uses logical reasoning in different ways to achieve entirely distinct results, but the core logical structure remains the same.

C. Embracing Mathematics and AI in Law

Lawyers are already harnessing the power of AI in their day-to-day work.[209] In using databases such as Westlaw and Lexis, for instance, lawyers are now trained in how to effectively use a search engine. What a lot of lawyers don't know is the workings of AI, which are happening in the background through search optimization. Further, lawyers have started to use newer technology such as draft assistant. AI is being used in six areas of legal practice: (1) electronic discovery, (2) "expertise automation," (3) research, (4) document management, (5) document creation and analytics for contracts and litigation, and (6) predictive analytics.[210]

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206. *Id*.

207. *Id*. at 12–13. *See also* PEDRO DOMINGOS, THE MASTER ALGORITHM: HOW THE QUESTION FOR THE ULTIMATE LEARNING MACHINE WILL REMAKE OUR WORLD 8 (2015) (describing machine learning as a subset of AI).

208. *What is Applied Mathematics?,* NW. UNIV. SCH. OF ENG'G, https://www.mccormick.northwestern.edu/applied-math/about/what-is-applied-mathematics.html [https://perma.cc/H4JG-TKZQ].

209. Westlaw promises faster research with the use of AI-powered tools. In using "state-of the-art" AI, Westlaw "provides attorneys with the fastest answers and most valuable insights via the next generation of legal search, intelligent document analysis, integrated ligation analytics, and the most powerful citator." *Westlaw Edge*, THOMSON REUTERS, https://legal.thomsonreuters.com/en/c/legal-research-westlaw-edge?searchid=TRPPCSOL/Google/LegalUS_RS_Westlaw_Main_Search_Brand-Phrase_US/Westlaw-Phrase&chl=ppc&cid=9029030&sfdccampaignid=7014O000001BRReQAO&ef_id=Cj0KCQiAkZKNBhDiARIsAPsk0WjW1Z8pjATDB7Uju9dN3P8QREvrJ1OvZo9mn-ZsHz5Nc496RJY0-soaAsekEALw_wcB:G:s&s_kwcid=AL!7944!3!417749241023!p!!g!!westlaw&gclid=Cj0KCQiAkZKNBhDiARIsAPsk0WjW1Z8pjATDB7Uju9dN3P8QREvrJ1OvZo9mn-ZsHz5Nc496RJY0-soaAsekEALw_wcB [https://perma.cc/PH3J-JW8T].

210. Anthony E. Davis, *The Future of Law Firms (And Lawyers) in the Age of Artificial Intelligence*, 27 THE PRO. LAW. 1 (2020), https://www.americanbar.org/groups/professional_responsibility/publications/professional_lawyer/27/1/the-future-law-firms-and-lawyers-the-age-artificial-intelligence/ [https://perma.cc/9K8N-NEHT].

Electronic discovery tools are widely used. These tools greatly reduce the amount of time and cost required to review discovery documents and produce results with greater accuracy.[211] AI first appeared in legal practice in electronic discovery.[212] These products use a concept called "conceptual searching" or "conceptual clustering" to search for relevant words as the tool reads the documents and then cluster the resulting information and documents.[213] The resulting data is provided visually in, for instance, a pie chart.[214] A person conducting the discovery needs to work with the algorithm and review the documents.[215] In other words, a human must synthesize and review the data, although the AI can narrow the field significantly.[216]

The concept of expertise automation refers to the ability of a lawyer or non-lawyer to use AI tools to engage with the law or mimic the role of a lawyer.[217] Examples of these products include Blue J, where a person using the product can assess issues of tax or employment law in a particular jurisdiction and input key facts of the matter at hand.[218] DoNotPay is a program that assists a person in contesting parking tickets anywhere in the U.S.[219] AI can assist a person in drafting wills or addressing housing issues.[220]

Today, lawyers conduct the majority of their research online.[221] Legal research has developed dramatically since the 1990s from the traditional library models to almost entirely digital platforms.[222] Search engines that lawyers use to conduct research rely upon AI. Westlaw and LexisNexis have both large databases and AI functions, which again allow a person conducting a search to efficiently comb sources, thereby again saving time and money for a client.[223]

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211. *Id.*; *Artificial Intelligence and Machine Learning in E-Discovery and Beyond,* DELOITTE, https://www2.deloitte.com/ch/en/pages/forensics/articles/AI-and-machine-learning-in-E-discovery.html [https://perma.cc/24B8-N4WY] (noting that legal professionals save 40% of hours typically needed to complete task).

212. Davis, *supra* note 210.

213. DELOITTE, *supra* note 211.

214. *Id.*

215. *Id.*

216. *Id.*

217. Davis, *supra* note 210.

218. *Id.*; BLUE J, *supra* note 9.

219. Davis, *supra* note 210; DONOTPAY, https://donotpay.com/learn/parking-tickets [https://perma.cc/SE53-EDKQ].

220. Davis, *supra* note 210.

221. Nicole Black, *ABA Report: Lawyers and Technology Trends in 2021*, ABOVE THE LAW (Aug. 26, 2021, 11:43 AM), https://abovethelaw.com/2021/08/aba-report-lawyers-and-technology-trends-in-2021/ [https://perma.cc/J2CM-QS8P].

222. Alarie, Niblet & Yoon, *supra* note 8, at 112.

223. Davis, *supra* note 210.

Westlaw Edge and LexisNexis use AI to enable users to search using natural language processing.[224] Even in conducting a simple Google search, a lawyer is inputting different variables and conditions and relying on the search engine to "rank" the results according to relevance.[225] In doing so, the lawyer is relying on AI and deep learning algorithms that continually optimize a lawyer's searching ability.[226]

Corporations use document management tools to create consistency and to enforce the contracts across all the contracts in the organization.[227] These tools can reduce time by thousands of hours.[228] AI is also being used to create contracts and litigation documents consistent with precedent.[229]

One form of predictive analytics allows a person to assess the likelihood of a judge rendering a particular decision on an issue based on the judge's past ruling on the same type of issue.[230] Lex Machina and Lex Predict are such examples.[231] Premonition can predict whether a particular lawyer will have success in front of a certain judge.[232] Another form of predictive analytics assists a lawyer in determining whether any key precedents have been excluded from a brief.[233]

This brief survey highlights the current functionality of AI in the legal profession.[234] We emphasize that these products are sophisticated tools, but they are just tools, nonetheless. In some instances, the tool can replace an attorney for a task for which, perhaps, a non-lawyer would have already declined representation, such as to contest a parking ticket or deal with a housing matter. As the complexity increases, however, lawyers must actively understand how to use and interpret the tools guided by AI. To do otherwise is

224. Michael Mills, *Artificial Intelligence in Law: The State of Play 2016*, THOMPSON REUTERS LEGAL EXEC. INST. (2016), https://britishlegalitforum.com/wp-content/uploads/2016/12/Keynote-Mills-AI-in-Law-State-of-Play-2016.pdf [https://perma.cc/M8W7-BBQ4].

225. Davide Negrini, Andrea Padoan & Mario Plebani, *Between Web Search Engines and Artificial Intelligence: What Side is Shown in Laboratory Tests?*, 8 DIAGNOSIS 227, 228 (2020).

226. *Id.*

227. Davis, *supra* note 210.

228. *Id.* (citing Hugh Son, *JPMorgan Software Does in Seconds What Took Lawyers 360,000 Hours*, BLOOMBERG (Feb. 27, 2017)), https://www.bloomberg.com/news/articles/2017-02-28/jpmorgan-marshals-an-army-of-developers-to-automate-high-finance [https://perma.cc/EY9X-4RAE]).

229. *Id.*

230. *Id.*

231. Mills, *supra* note 224.

232. *Id.*

233. Davis, *supra* note 210.

234. For further detailed reading, see KEVIN D. ASHLEY, ARTIFICIAL INTELLIGENCE AND LEGAL ANALYTICS: NEW TOOLS FOR LAW PRACTICE IN THE DIGITAL AGE (2017).

professional incompetence and might result in malpractice or professional responsibility issues.[235] The tool does not replace the lawyer.

Cass Sunstein has argued that AI cannot replace legal reasoning.[236] We agree. But lawyers need to understand the interplay, the point of connection, between their reasoning and the algorithms. Where do the results of the algorithmic programs end and their own strategy and understanding of the complexity of the law as applied to their client's situation begin? How can they use their human reasoning and skill to oversee the tools they are using? Lawyers can use the metaphor of mathematics as an analog to legal reasoning to help themselves, and their clients, understand what they as attorneys offer. Lawyers need the metaphor to grasp their professional responsibilities in a world where pen and paper are supplemented, and sometimes supplanted, by code and computer.

VI. RECOMMENDATIONS AND CONCLUSION

We need to move past our discomfort with mathematics. For one thing, lawyers' discomfort with mathematics is leading to meritless lawsuits and unwarranted litigation.[237] In dismissing a case over a three-week lapse in blood pressure medication, the Seventh Circuit Court of Appeals chastised the judges and lawyers involved in the case for failing to support adequate medical evidence to support the inmate's claimed symptoms.[238] In his opinion, Judge Posner observed that "[t]he discomfort of the legal profession, including the judiciary, with science and technology is not a new phenomenon . . . [b]ut it's increasingly concerning, because of the extraordinary rate of scientific and other technological advances that figure increasingly in litigation."[239]

235. Victoria Hudgins, *Lawyers Run into Legal, Ethical Risks Using—and Not Using—AI Technology*, LEGALTECH NEWS (Oct. 14, 2021, 11:30 AM), https://www.law.com/legaltechnews/2021/10/14/lawyers-run-into-legal-ethical-risks-using-and-not-using-ai-technology/ [https://perma.cc/33VB-4E64].

236. Cass R. Sunstein, *Of Artificial Intelligence and Legal Reasoning*, 8 UNIV. OF CHI. ROUNDTABLE 29, 31 (2001), https://chicagounbound.uchicago.edu/cgi/viewcontent.cgi?article=12376&context=journal_articles [https://perma.cc/8DVR-7SFN].

237. Debra Cassens Weiss, *Posner: Lawyers Bad at Math are an Increasing Concern; Inmate's Blood-Pressure Suit Shows Why*, A.B.A. JOURNAL (Oct. 29, 2013, 12:51 PM), https://www.abajournal.com/news/article/posner_math_block_lawyers_an_increasing_concern_inmates_blood-pressure_suit [https://perma.cc/9THP-MCR9]; *Should Lawyers Learn to Code?*, THE TEMPLE 10-Q, https://www2.law.temple.edu/10q/should-lawyers-learn-to-code/ [https://perma.cc/S8FS-BDSH] (arguing for the need of lawyers competent in STEM fields as "[t]here is a long list of examples of questionable policies, legislation, and rulemaking in law and technology that can also be traced to these fundamental misunderstandings").

238. *Jackson v. Pollion*, 733 F.3d 786, 787–88 (7th Cir. 2013).

239. *Id.* at 788.

Specifically, Judge Posner focused on the fact that innumerable lawyers pursued the law instead of STEM due to a "math block" and generally law students display a peculiarly adverse reaction to math and science.[240]

For another thing, and more importantly, AI is demonstrating that mathematics and law do co-exist in harmony. While mathematics and law are distinct disciplines, mathematical algorithms are being used to create faster research tools and even predictive analysis. Lawyers should not ignore that connection. While human lawyering requires more than research and analysis of legal standards, lawyers should not shy away from understanding the mathematics underlying the tools we will increasingly use.[241] Lawyers should also see an analog between mathematical thinking and legal reasoning, including that mathematics is syntactic.[242] We must therefore continue to sharpen our abilities to think logically and formulate logical arguments. Using good logic is fundamental to lawyering, but it is not the only criteria for good lawyering.

We recommend that law schools promote legal education to mathematics students. In doing so, law schools should discuss how mathematical reasoning parallels legal reasoning. The legal profession would gain diversity of thought by bringing in more students who have studied mathematics. Those students do not need to consider solely intellectual property areas of law: their skills would benefit all practice areas. Additionally, undergraduate institutions should actively promote that idea to prelaw students that they should be taking classes in logic and mathematics.

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240. *Id.* (citing DAVID L. FAIGMAN, MICHAEL SAKS, JOSEPH SANDERS & EDWARD CHENG, MODERN SCIENTIFIC EVIDENCE: STANDARDS, STATISTICS, AND RESEARCH METHODS, at v (Student ed. 2008)).

241. There is an ongoing debate in the mathematical community surrounding Gödel's incompleteness theorems as they relate to the essential question of what makes us human. GOLDSTEIN, *supra* note 65, at 201–03. In understanding this debate, lawyers can improve upon the human aspect of legal practice.

Gödel's incompleteness theorems provide that "[i]n any formal system adequate for number theory there exists an undecidable formula—that is, a formula that is not provable and whose negation is not provable," and that "the consistency of a formal system adequate for number theory cannot be proved within the system." *Id.* at 23. These theorems have raised the debate of whether humans are machines or whether "there must be more to human thinking than can ever be achieved by a computer, in the sense that we understand the term 'computer' today." *Id.* at 201–03 (quoting ROGER PENROSE, SHADOWS OF THE MIND: A SEARCH FOR THE MISSING SCIENCE OF CONSCIOUSNESS 65 (1994)).

This debate is mirky and complicated. As a practical takeaway, this debate highlights the importance of lawyers' ability to reason logically, to "think like a machine," and to draw upon rhetoric, to get at the heart of what makes us human. While we may never know exactly what makes us human, rhetoric requires us to draw upon customs, policy, and sound judgment.

242. *Id.* at 100 (noting that "[m]athematics, like logic, is syntactic").

Specifically, law students and lawyers should seek to increase their understanding of numeracy, their "ability to understand and use numbers."[243] Numeracy both demonstrates the connection between legal thinking and mathematical reasoning and reveals lawyers' ability to think mathematically, despite the pervasive view that lawyers are bad at math. An empirical study on numeracy and legal decision making suggests that mathematical thinking impacts a lawyer's ability to conduct substantive legal analysis.[244] Its findings demonstrate the need to think of mathematical reasoning beyond mere arithmetic, and instead to encompass decisions that include the ability to evaluate probabilities, risks, or calculations.[245]

We further recommend that law school classes discuss logic in classroom dialogue and exercises to sharpen students' legal reasoning. The logic games portion of the LSAT should not be the last or only time students are exposed to logic. Students should understand and be able to identify logical fallacies and know how to argue against them.[246] They should know how to appropriately set up arguments based on deductive reasoning, analogy, and counterargument. To foster this learning, students should be exposed to the works of philosophers and rhetoricians who have considered the relationship between logic, rhetoric, and legal argument.

Moreover, students should understand the tools of legal research, analysis, and drafting available on the market. Not only should they be aware that they exist and how to use them, but they should have some understanding of how AI drives the tools. That understanding will be increasingly important as a matter of professional competency. Attorneys will need to understand how the tools work so they can see gaps in the research and analysis. The AI tools should be a starting point, not necessarily an ending point.

While this philosophical and mathematical debate is important, the practical matter for lawyers is that AI has emerged in the practice of law. AI is the future of the legal field, ready or not. While the legal field is usually slow to adapt to changes, we, as lawyers, have a unique opportunity to embrace the rapid changes that are coming and harness the power of AI. To do so, we need to

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243. Rowell & Bregant, *supra* note 2, at 197. *See also* James Ming Chen, *Legal Quanta: A Mathematical Romance of Many Dimensions*, 2016 MICH. ST. L. REV. 313, 314 (2016) ("Innumeracy is flatly unacceptable.").

244. Rowell & Bregant, *supra* note 2, at 226.

245. *Id.*

246. In 2007, Ruggero Aldisert and two of his clerks wrote that they find it tragic that law schools fail to teach logic. Ruggero J. Aldisert, Stephen Clowney & Jeremy D. Peterson, *Logic for Law Students: How to Think Like a Lawyer*, 69 U. PITT. L. REV. 1, 2 (2007) ("The failure to ground legal education in principles of logic does violence to the essence of the law. Leaving students to distill the principles of logic on their own is like asking them to design a rocket without teaching them the rules of physics.").

understand mathematical thinking. But understanding mathematical thinking will serve us in more ways than just using AI—it will make us better thinkers, writers, and overall better lawyers.